FUZZY IDEALS IN ORDERED SEMIGROUPS AND THEIR GENERALISATIONS

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TO MY BELOVED FAMILY ESPECIALLY MY FATHER

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ABSTRACT

The idea of fuzzy sets has opened a new era of research in the world of contemporary mathematics. The proposed concept of fuzzy sets provided for a renewed approach to model imprecision and uncertainty present in phenomena without sharp boundaries. The fuzzification of algebraic structures, particularly ordered semigroups, play a prominent role in mathematics with diverse applications in many applied branches such as computer arithmetic, control engineering, error-correcting codes and formal languages. In this background, many researchers initiated the notion of "quasi coincident with" (q) relation between a fuzzy point and a fuzzy set in ordered semigroups. Later a new generalisation of quasi-coincident with relation symbolised as q_k where $k \in [0,1)$ has been introduced. In this thesis, new concepts including fuzzy ideals, fuzzy interior ideals, fuzzy generalised bi-ideals, fuzzy bi-ideals and fuzzy quasi-ideals of type $(\in, \in \vee q_k)$ of ordered semigroup are introduced. Further, ordinary ideals and $(\in, \in \vee q_k)$-fuzzy ideals are linked using level subset and characteristic function. The results show that in regular, intra-regular and semisimple ordered semigroups both $(\in, \in \vee q_k)$-fuzzy ideals and $(\in, \in \vee q_k)$-fuzzy interior ideals coincide. The concept of upper/lower parts of $(\in, \in \vee q_k)$-fuzzy interior ideals is also introduced and furthermore, semisimple, simple and intra-regular ordered semigroups are characterised in terms of this notion. The relation between generalised bi-ideals and $(\in, \in \vee q_k)$-fuzzy generalised bi-ideals is determined. Furthermore, the conditions for the lower part of $(\in, \in \vee q_k)$-fuzzy generalised bi-ideal to be a constant function are provided. The characterisations of ordered semigroups by the properties of semiprime $(\in, \in \vee q_k)$-fuzzy quasi-ideals are investigated. Finally, the classification of ordered semigroups by $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy interior ideals and $(\overline{\in_\gamma}, \overline{\in_\gamma} \vee \overline{q_\delta})$-fuzzy interior ideals are determined comprehensively.

ABSTRAK

Idea mengenai set kabur telah membuka suatu era baharu bagi penyelidikan dalam dunia matematik sezaman. Cadangan konsep set kabur diketengahkan untuk pembaharuan pendekatan kepada ketidaktepatan dan ketakpastian yang wujud dalam fenomena tanpa sempadan yang tepat. Pengaburan bagi struktur aljabar, khasnya bagi semikumpulan bertertib, memainkan peranan utama dalam matematik dengan pelbagai aplikasi dalam banyak cabang gunaan seperti aritmetik komputer, kejuruteraan kawalan, kod pembetulan-ralat dan bahasa formal. Dengan latar belakang ini, ramai penyelidik memulakan konsep "kuasi-kebetulan dengan" hubungan (q) antara satu titik kabur dengan satu set kabur dalam semikumpulan bertertib. Kemudian, satu pengitlakan bagi kuasi-kebetulan dengan hubungan yang ditandakan sebagai q_k di mana $k \in [0,1)$ diperkenalkan. Dalam tesis ini, konsep baharu termasuk unggulan kabur, unggulan pedalaman kabur, dwi-unggulan kabur teritlak, dwi-unggulan kabur dan kuasi-unggulan kabur dengan jenis bagi semikumpulan bertertib diperkenalkan. Selanjutnya, unggulan biasa dan unggulan kabur-$(\in, \in \vee q_k)$ dikaitkan menggunakan subset aras dan fungsi cirian. Keputusan menunjukkan bahawa dalam semikumpulan bertertib sekata, intra-sekata dan semi-ringkas, kedua-dua unggulan kabur-$(\in, \in \vee q_k)$ dan unggulan pedalaman kabur-$(\in, \in \vee q_k)$ adalah sama. Konsep bahagian atas/bawah bagi unggulan pedalaman kabur-$(\in, \in \vee q_k)$ juga diperkenalkan dan seterusnya, semikumpulan bertertib semi-ringkas, ringkas dan intra-sekata dicirikan berdasarkan konsep ini. Hubungan antara dwi-unggulan teritlak dengan dwi-unggulan kabur teritlak-$(\in, \in \vee q_k)$ telah ditentukan. Sebagai tambahan, syarat-syarat bagi bahagian bawah dwi-unggulan kabur teritlak-$(\in, \in \vee q_k)$ untuk menjadi fungsi malar diberikan. Pencirian bagi semikumpulan bertertib menggunakan sifat-sifat semiperdana kuasi-unggulan kabur-$(\in, \in \vee q_k)$ telah dikaji. Akhir sekali, pengelasan bagi semikumpulan bertertib oleh unggulan pedalaman kabur-$(\in_\gamma, \in_\gamma \vee q_\delta)$ dan unggulan pedalaman kabur-$(\overline{\in_\gamma}, \overline{\in_\gamma} \vee \overline{q_\delta})$ ditentukan secara menyeluruh.